Week 6

Difficulty level*is 1

Given $f_T(t) = \alpha e^{-\alpha t}$ for $t \geq 0, \alpha > 0$..

Find m(x), the median of T(x).

A) $\frac{1}{\alpha}\ln(2)$

B) $\alpha \ln(2)$

C) $\frac{1}{\alpha}\ln\left(\frac{1}{2}\right)$

D) $\frac{1}{\alpha}$

E) $\ln\left(\frac{1}{\alpha}\right)$

Week 5

Difficulty level*is 2

Given

i. $l_{50} = 980$

ii. $l_{51} = 960$

iii. $l_{52} = 930$

iv. $l_{53} = 890$

Rank $_{3}q_{50}$, $_{2}q_{51}$, and $_{1|2}q_{50}$ from smallest to largest.

A) III < II < I
B) I < II < III
C) I < III < II
D) II < I < III
E) III < I < II

*ADAPT questions have a difficulty definition on a scale from 1 to 10 with 1 representing easy and 10 representing very difficult.

Week 4

Difficulty level*is 1

Given $s(x)=\frac{3}{1+2x}$, find m(z), the median future lifetime of a person aged (z).

A) 0.5+z
B) 0.5
C) 0.5z
D) 2z
E) 2+z

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Week 3

Difficulty level*is 4

Mortality table A has a force of mortality μ_{x+t}^{A} and mortality rate q_x^{A}.

Mortality table B has a force of mortality μ_{x+t}^{B} and mortality rate q_x^{B}.

You know that $\mu_{x+t}^{B} = 0.25\,\mu_{x+t}^{A}$ for $0 \le t \le 1$. Find q_x^{B}.

A) $1-(1-q_x^{A})^{1/4}$

B) $(1-q_x^{A})^{1/4}$

C) $1-(1-q_x^{A})^{1/2}$

D) $(q_x^{A})^{1/4}$

E) $1-(q_x^{A})^{1/4}$

Week 2

Difficulty level*is 4

Given $\mu_x = \dfrac{3x^2}{1000 - x^3}$ for $0 \le x \le 10$, find q_5.

A) $\dfrac{13}{125}$

B) $\dfrac{13}{25}$

C) $\dfrac{31}{1000}$

D) $\dfrac{13}{875}$

E) $\dfrac{31}{125}$

Week 1

Difficulty level*is 2

Given the survival function $S(x) = \frac{\sqrt{100-x}}{25}$ for $0 \leq x \leq 100$, calculate $F(75), f(75)$, and μ_{75}.

A) $F(75) = 0.8, f(75) = 0.004,\ \mu_{75} = 0.02$

B) $F(75) = 0.8, f(75) = 0.04,\ \mu_{75} = 0.01$

C) $F(75) = 0.5, f(75) = 0.004,\ \mu_{75} = 0.01$

D) $F(75) = 0.5, f(75) = 0.4,\ \mu_{75} = 0.01$

E) $F(75) = 0.8, f(75) = 0.04,\ \mu_{75} = 0.02$

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Week 6

$m(x)$ is defined such that $\frac{x(x+m(x))}{s(x)}=\frac{1}{2}$

$$F(x)=\int_0^x f(t)dt=1-e^{-\alpha x}$$

$$s(x)=1-F(x)=e^{-\alpha x}$$

$$\frac{e^{-\alpha(x+m(x))}}{e^{-\alpha x}}=\frac{1}{2}\Rightarrow e^{-\alpha m(x)}=\frac{1}{2}$$

$$m(x)=\frac{1}{\alpha}\ln(2)$$

Week 5

$$\text{Let } I = {}_{3}q_{50} = \frac{980-890}{980} = \frac{9}{98}$$

$$II = {}_{2}q_{51} = \frac{960-890}{960} = \frac{7}{96}$$

$$III = {}_{1|2}q_{50} = \frac{960-890}{980} = \frac{7}{98}$$

$$\therefore III < II < I\,.$$

Week 4

$$m(z)\,p_z = \frac{s(x+m(z))}{s(z)} = \frac{\dfrac{3}{1+2(z+m(z))}}{\dfrac{3}{1+2z}} = \frac{1+2z}{1+2(z+m(z))} = \frac{1}{2}$$

$$\Rightarrow m(z) = 0.5 + z$$

Week 3

$$p_x^A = e^{-\int_0^1 \mu_{x+t}^A \, dt}$$

$$p_x^B = e^{-\int_0^1 \mu_{x+t}^B \, dt} = e^{-\int_0^1 0.25 \mu_{x+t}^A \, dt} = \left(p_x^A\right)^{1/4}$$

$$q_x^B = 1 - p_x^B = 1 - \left(p_x^A\right)^{1/4} = 1 - \left(1 - q_x^A\right)^{1/4}$$

Week 2

$${}_tp_0 = e^{\int_0^t \mu_s ds} = e^{-\int_0^t (3s^2)/(1000-s^3)ds}$$

Letting $u = s^3$, and $du = 3s^2 ds$,

$${}_tp_0 = e^{-\int_0^{t^3} 1/(1000-u)du} = e^{\ln(1000-u)|_{u=0}^{t^3}} = \frac{1000-t^3}{1000}$$

$$\therefore q_5 = 1 - \frac{{}_6p_0}{{}_5p_0} = 1 - \frac{1000-(6)^3}{1000-(5)^3} = \frac{13}{125}$$

Week 1

$$F(x) = 1 - S(x) = \frac{25 - \sqrt{100 - x}}{25} \Rightarrow F(75) = 1 - \frac{\sqrt{25}}{25} = 0.8$$

$$f(x) = F'(x) = \frac{d}{dx}\left(1 - \frac{1}{25}(100 - x)^{1/2}\right) = \frac{1}{50\sqrt{100 - x}}$$

$$\Rightarrow f(75) = \frac{1}{250} = 0.004$$

$$\mu_{75} = \frac{f(75)}{1 - F(75)} = \frac{0.004}{1 - 0.8} = 0.02$$