Week 8

Difficulty level*is 8.6066465

Given $P(A|B) = .5$, $P(B|A) = .4$,

and $P(B|A^C) = .3$.

Determine $P(A)$.

A) 0.43
B) 0.27
C) 0.21
D) 0.50
E) 0.35

*ADAPT questions have a difficulty definition on a scale from 1 to 10 with 1 representing easy and 10 representing very difficult.

Week 7

Difficulty level*is 5.6627346

Let X be a continuous random variable with density function

$$f(x) = \begin{cases} 0.4|x-1| & 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

Compute $E(X)$.

A) 1.3
B) 1.6
C) 1.8
D) 1.9
E) 2.2

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Week 6

Difficulty level*is 6.1404183

The cost of a week long vacation in a city is 1000 with a variance of 1200. The city imposes a tax which will raise all tourism related costs by 10%. Find the coefficient of variation for the cost of a week long vacation in this city after the tax is imposed.

A) 0.03
B) 0.35
C) 0.63
D) 0.85
E) 1.21

*ADAPT questions have a difficulty definition on a scale from 1 to 10 with 1 representing easy and 10 representing very difficult.

Week 5

Difficulty level*is 6.9543400

A city accepts the minimum of three bids on a construction project. The three bids are independently and uniformly distributed on the interval from 100,000 to 110,000. Find the expected value of the accepted bid.

A) 100,500
B) 101,500
C) 102,500
D) 103,500
E) 104,500

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Week 4

Difficulty level*is 9.3840573

The dread disease brainfreezitis afflicts 2% of actuarial students taking an exam. Two tests are developed to detect this affliction. Test 1 gives positive results 98% of the time for an afflicted person and 4% of the time for a non-afflicted person. Test 2 gives positive results 99% of the time for an afflicted person and 6% of the time for a non-afflicted person. The outcome of each test is independent of the other test. If a randomly selected student tests positive with both tests, what is the probability that student actually has brainfreezitis?

A) 0.89
B) 0.97
C) 0.84
D) 0.80
E) 0.74

Week 3

Difficulty level*is 7.3771884

Claim amounts are independently and uniformly distributed on [0, 1000]. Find the expected value of the difference between the largest and smallest of 3 randomly selected claims.

A) 667
B) 500
C) 300
D) 229
E) 333

*ADAPT questions have a difficulty definition on a scale from 1 to 10 with 1 representing easy and 10 representing very difficult.

Week 2

Difficulty level*is 8.7866066

The number of tornadoes in a county during a year is modeled with the following assumptions:

In a calendar year the number of tornadoes is binomial with $n = 2$ and $p = 0.01$.

The number of tornadoes in any year is independent of any other year.

Under these assumptions calculate the probability the county will experience exactly 2 tornadoes in a ten year period.

A) 0.004
B) 0.016
C) 0.028
D) 0.057
E) 0.100

Week 1

Difficulty level*is 4.8344553

A teacher teaches two classes with 8 students each. Each student has a 95% chance of passing their class independent of the other students.

Find the probability all 8 students pass in exactly one of the two classes.

A) 0.45
B) 0.66
C) 0.38
D) 0.33
E) 0.32

*ADAPT questions have a difficulty definition on a scale from 1 to 10 with 1 representing easy and 10 representing very difficult.

SALT SOLUTIONS

Week 8

We know:

$$\frac{P(A \text{ and } B)}{P(B)} = 0.5$$
$$\frac{P(A \text{ and } B)}{P(A)} = 0.4$$

So,

$$P(A \text{ and } B) = 0.5P(B) = 0.4P(A) \text{ so, } P(B) = 0.8P(A) \text{ .}$$

We are also given:

$$\frac{P(B \text{ and } A^C)}{P(A^C)} = 0.3.$$

But,

$$\frac{P(B \text{ and } A^C)}{P(A^C)} = \frac{P(B) - P(B \text{ and } A)}{1 - P(A)}$$
$$= \frac{0.8P(A) - 0.4P(A)}{1 - P(A)} = \frac{0.4P(A)}{1 - P(A)} \text{ .}$$

Now we can set this equal to 0.3 and solve for $P(A)$.

$$\frac{0.4P(A)}{1 - P(A)} = 0.3$$
$$0.4P(A) = 0.3(1 - P(A))$$
$$0.4P(A) = 0.3 - 0.3P(A)$$
$$0.7P(A) = 0.3$$
$$P(A) = \frac{3}{7} = 0.4286$$

Week 7

$E(X) = \int xf(x)dx$

$E(X) = \int_0^3 .4x\left|x - 1\right|dx$

$E(X) = \int_1^3 (.4x^2 - .4x)dx + \int_0^1 (.4x - .4x^2)dx$

$= (3.6 - 1.8) - (.1333 - .2) + (.2 - .1333) = 1.8 + .0667 + .0667$

$= 1.9334$

Week 6

Coefficient of variation = $\frac{\sigma}{\mu}$

Let O = old cost

Let N = new cost

$$E(O) = 1000$$
$$E(N) = E(1.1(O)) = 1100$$
$$Var(O) = 1200$$
$$Var(N) = Var(1.1(0)) = 1.1^2(1200) = 1452$$
$$\sigma_N = \sqrt{Var(N)} = \sqrt{1452} \approx 38.1051$$
$$\text{Coefficient of variation} = \frac{\sigma}{\mu} = \frac{38.1051}{1100} = .03464$$

Week 5

Let Y = accepted bid value

We need to find $P(Y \leq y)$ which equals $1 - P(Y > y)$. For this particular problem, this is equal to $1 - P(X > y)^3$ (as we are finding the probability that ALL three bids are greater than y.

Next, we want to substitute $\frac{110,000 - y}{10,000}$ in for $P(X > y)$. This gives

$$1 - \left(\frac{110,000 - y}{10,000}\right)^3$$

which is the cumulative function. However, we need the density function which is simply the derivative of the cumulative function.

For ease when using the calculator, let's finish the problem in 1000's

Next, we have to integrate to find the expected value

$$\frac{3}{1000}\int_{100}^{110} y(110 - y)^2 dy$$
$$\frac{3}{1000}\int_{100}^{110} 12100y - 220y^2 + y^3 dy$$

$$\frac{3}{1000}\left[6050y^2 - \frac{220y^3}{3} + \frac{y^4}{4}\right]\Bigg|_0^{110}$$

$$\frac{3}{1000}[(73,205,000 - 97,606,666.67 + 36,602,500) - (60,500,000 - 73,333,333.33 - 25$$

$$= \frac{3}{100}[34,166.67] = 102.49998$$

Remember, we divided everything by 1000 in the beginning to make using the calculator easier. Now we have to multiply by 1000 to solve for the actual bid.

$$102.49998(1000) = 102,499.98$$

Note: the slight difference is due to rounding

Week 4

Let,

$A =$ student has brainfreezitis
$B =$ first test is positive
$C =$ second test is positive

$$P(A|B \cap C) = \frac{P(A)P(B \cap C|A)}{P(A)P(B \cap C|A) + P(A^C)P(B \cap C|A^C)}$$

$$= \frac{P(A)P(B|A)P(C|A)}{P(A)P(B|A)P(C|A) + P(A^C)P(B|A^C)P(C|A^C)}$$

$$= \frac{(0.02)(0.98)(0.99)}{(0.02)(0.98)(0.99) + (0.98)(0.04)(0.06)}$$

$$= 0.892$$

Week 3

For the minimum:

Let Y =the minimum of the 3 claims X_1 , X_2 and X_3 in thousands.

Let Z =the minimum of the 3 claims X_1 , X_2 and X_3 in thousands.

We need to find $P(Y \leq y)$ which equals $1 - P(Y > y)$. For this particular problem, this is equal to $1 - P(X_i > y)^3$ (as we are finding the probability that ALL 3 bids are greater than y. Next we want to substitute $1 - y$ in for $P(X > y)$. This gives

$$1 - (1 - y)^3$$

which is the cumulative function. However, we need the density function which is simply the derivative of the cumulative function

$$\frac{d}{dx}[1 - (1 - y)^3] = -3(1 - y)^2(-1)$$

Next, we have to integrate to find the expected value

$$\begin{aligned}
&\int_0^1 3y(1 - y)^2 dy \\
&= \int_0^1 3y(1 - 2y + y^2)dy \\
&= \int_0^1 (3y - 6y^2 + 3y^3)dy \\
&= \left[\frac{3y^2}{2} - 2y^3 + \frac{3y^4}{4}\right]\Bigg|_0^1 \\
&= \frac{3}{2} - 2 + \frac{3}{4} = .25
\end{aligned}$$

Since we were working in 1000s, this equals 250

Since the original distributions were uniform, the maximum will be 250 from the top hence the expectation of the maximum is
$E(Z) = 1000 - 250 = 750$

(Note: recognizing when a symmetry exists in a problem can save you valuable time)

So,

$$E(Z - Y) = E(Z) - E(Y) = 750 - 250 = 500$$

Week 2

Since the number of tornadoes in each year is independent of other years and all are identical binomial :

Let Y = # of tornadoes in 10 years

Y is binomial with $n = 20$ and $p = 0.01$

We want:

$P(Y = 2) = \binom{20}{2}(0.01)^2(0.99)^{18} = .01586$

Week 1

The chance of all students passing in one class is $(.95)^8 = .6631$. So let

- A = event that all students pass in class 1
- B = event that all students pass in class 2

$$P[(A \cap B^c) \cup (A^c \cap B)]$$
$$= (.6631)(1 - .6631) + (1 - .6631)(.6631) = .4468$$